PRINCIPLES OF ENGINEERING ECONOMY

EIGHTH EDITION

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1-1. You are a college student in your final year. You have been forced by a housing shortage to take a room eight miles from your campus. You can ride a public bus to and from your campus; this travels at 40-minute intervals from 6 A.M. to 8 P.M. An alternative is to buy an automobile for your transportation. If you buy a car you expect to dispose of it at the end of the school year. What prospective cash disbursements and receipts seem to be relevant to your decision of whether or not to buy a car? What irreducibles do you think are important?

EXAMPLE 3-1

If $1,000 is invested at 6% compounded interest on January 1, 1990, how much will be accumulated by January 1, 2000? (Figure 3-2a.)

Solution:

$i = 0.06$; $n = 10$; $P = \$1{,}000$; $F = ?$

$$F = P(F/P, 6\%, 10)$$
$$= \$1{,}000(1.7908) = \$1{,}791$$

EXAMPLE 3-2

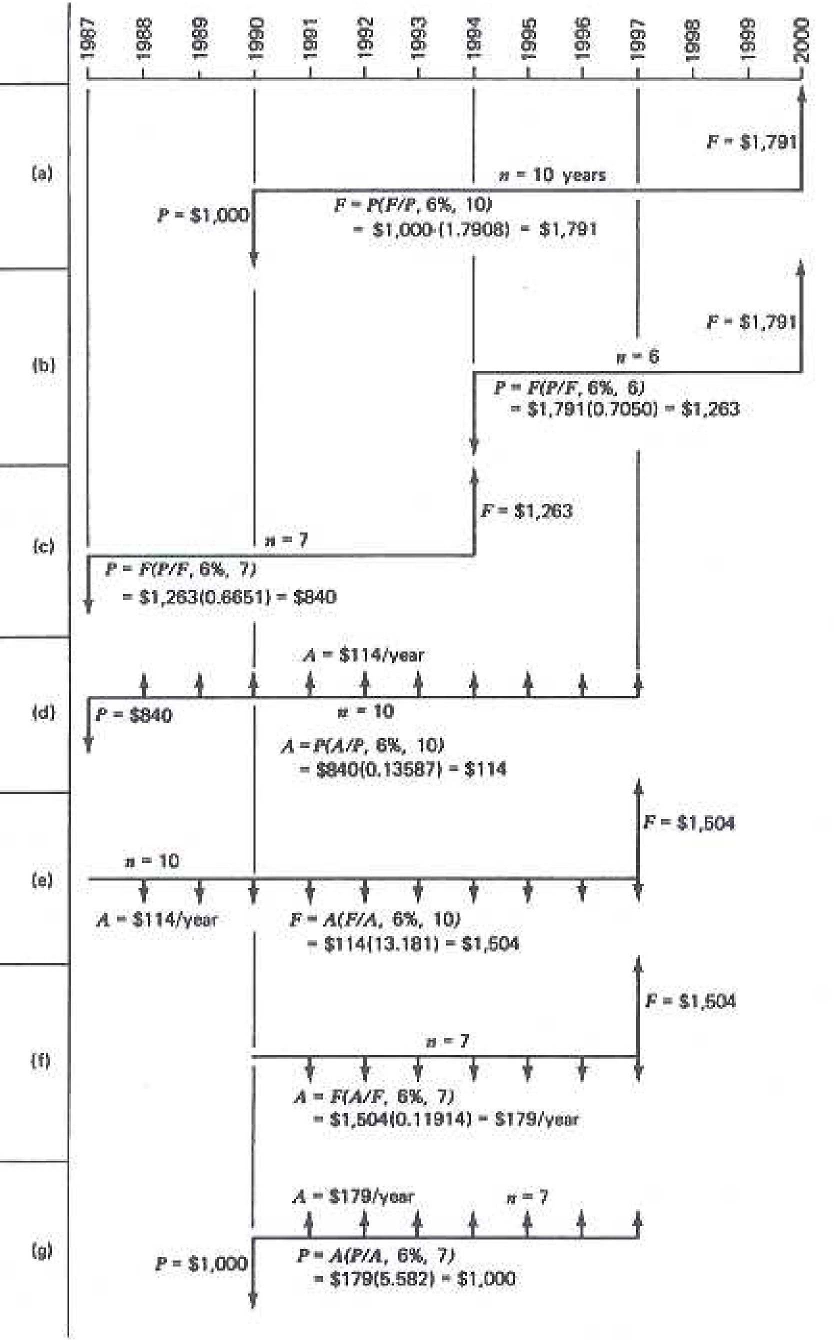
How much would you have to invest at 6% interest on January 1, 1994, in order to accumulate $1,791 on January 1, 2000? (Figure 3-2b.)

Solution:

$i = 0.06$; $n = 6$; $F = \$1{,}791$; $P = ?$

In this case zero time is assumed to be January 1, 1994.

$$P = F(P/F,6\%,6) = \$1{,}791(0.7050)$$
$$= \$1{,}263$$



1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
(a)
F = $1,791
n = 10 years
P = $1,000
F = P(F/P, 6%, 10)
= $1,000 (1.7908) = $1,791
(b)
F = $1,791
n = 6
P = F(P/F, 6%, 6)
= $1,791(0.7050) = $1,263
(c)
F = $1,263
n = 7
P = F(P/F, 6%, 7)
= $1,263(0.6651) = $840
(d)
A = $114/year
P = $840
n = 10
A = P(A/P, 6%, 10)
= $840(0.13587) = $114
(e)
F = $1,504
n = 10
A = $114/year
F = A(F/A, 6%, 10)
= $114(13.181) = $1,504
(f)
F = $1,504
n = 7
A = F(A/F, 6%, 7)
= $1,504(0.11914) = $179/year
(g)
A = $179/year
n = 7
P = $1,000
P = A(P/A, 6%, 7)
= $179(5.582) = $1,000

EXAMPLE 3–3

What is the present worth on January 1, 1987, of $1,263 on January 1, 1994, if interest is at 6%? (Figure 3–2c.)

Solution:

$i = 0.06$; $n = 7$; $F = \$1{,}263$; $P = ?$

Now zero time is assumed to be January 1, 1987.

$$P = F(P/F, 6\%, 7) = \$1{,}263(0.6651)$$
$$= \$840$$

EXAMPLE 3-4

If \$840 is invested at 6% on January 1, 1987, what equal year-end withdrawals can be made each year for 10 years to leave nothing in the fund after the tenth withdrawal? (Figure 3-2d.)

Solution:

$i = 0.06$; $n = 10$; $P = \$840$; $A = ?$

$$A = P(A/P, 6\%, 10) = \$840(0.13587)$$
$$= \$114.1$$

EXAMPLE 3-5

How much will be accumulated in a fund, earning 6% interest, at the end of 10 years if \$114.1 is deposited at the end of each year for 10 years, beginning in 1987? (Figure 3-2e.)

Solution:

$i = 0.06$; $n = 10$; $A = \$114.1$; $F = ?$

$$F = A(F/A,6\%,10) = \$114.1(13.181)$$
$$= \$1{,}504$$

EXAMPLE 3–6

How much must be deposited at 6% each year for 7 years beginning on January 1, 1991 in order to accumulate $1,504 on the date of the last deposit, January 1, 1997? (Figure 3–2f.)

Solution:

$i = 0.06$; $n = 7$; $F = \$1{,}504$; $A = ?$

Now zero date has returned to January 1, 1990.

$$A = F(A/F,6\%,7) = \$1{,}504(0.11914)$$
$$= \$179.2$$

EXAMPLE 3-7

How much would you need to deposit at 6% on January 1, 1990, in order to draw out \$179.2 at the end of each year for 7 years, leaving nothing in the fund at the end? (Figure 3–2g.)

Solution:

$i = 0.06$; $n = 7$; $A = \$179.2$; $P = ?$

$$P = A(P/A,6\%,7) = \$179.2(5.582)$$
$$= \$1{,}000$$

Formulas for Minimum-Cost Point

Example 10–3 dealt with a case in which cost varied with a certain variable of design, namely, pipe diameter. Some elements of cost increased and others decreased with an increase in the value of the design variable. In this common type of case there is presumably some value of the design variable that makes the sum of all costs a minimum.

Wherever the variation of cost as a function of a design variable can be expressed by an algebraic equation, it is possible to use calculus to find the value of the design variable that results in minimum cost. Over the years an entire field of scientific literature has developed around the simple model illustrated in the following paragraphs. It is referred to as inventory control theory, but it might more properly be called economic inventory management.[2]

The simplest case is one in which one element of cost varies in direct proportion to the variable of design, a second element of cost varies inversely as the variable of design, and all other costs are independent of this variable. Although minimum-cost point formulas may, of course, be developed for situations much more complex than this, some of those that have traditionally been used by engineers did actually deal with situations of this type. A general solution of the problem of finding the minimum-cost point in such circumstances follows.

Let y = total cost and let x = the variable of design. The situation of cost variation just described may be expressed by the equation $y = ax + \frac{b}{x} + c$

Taking the first derivative, we find

$$\frac{dy}{dx} = a - \frac{b}{x^2}$$

Equating this to zero, and solving for x,

$$x = \sqrt{\frac{b}{a}}$$

This is the value of the design variable that makes cost a minimum.

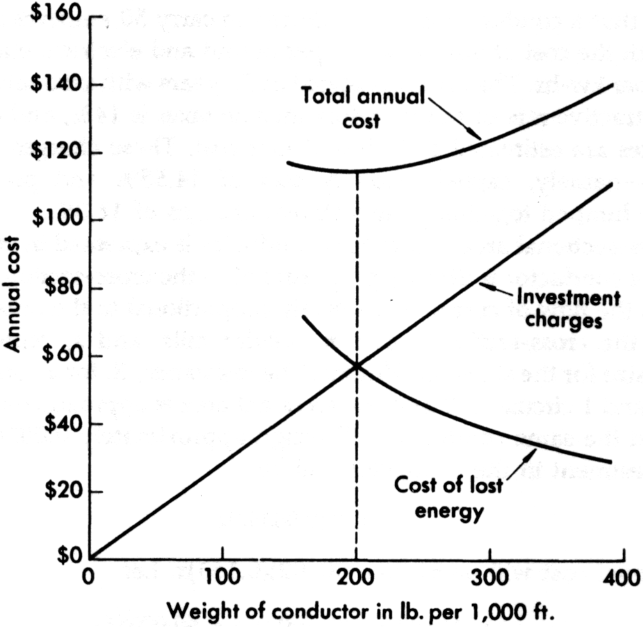
When $x = \sqrt{\frac{b}{a}}$, the directly varying costs equal the inversely varying costs. This fact is illustrated in Figure 10–1 and may be demonstrated as

$$ax = a\sqrt{\frac{b}{a}} = \sqrt{ab}; \frac{b}{x} = \frac{b}{\sqrt{\frac{b}{a}}} = \sqrt{ab}$$

The formula $x = \sqrt{\frac{b}{a}}$ can be applied to different kinds of problems, but it

[2]The primary emphasis in this text is the presentation of the basic principles and techniques of capital expenditure analysis. Since inventory and lot-size decisions are very complex and technical operating decisions, it would be impossible to present a definitive exposition of the field here. However, since the basic principles discussed in Chapter 1 apply to both types of decisions, and, as Example 10–4 illustrates, since the general formulation may be applied to investment decisions of certain types, a brief discussion of both the advantages and disadvantages of such formulas is desirable at this point.

Most texts on production and inventory control contain extensive developments of economic production quantity and purchase quantity models. One excellent example is G. Hadley and T. M. Whitin, *Analysis of Inventory Systems* (Englewood Cliffs, N.J.: Prentice–Hall, Inc., 1963).


$160
$140
$120
$100
$80
$60
$40
$20
$0
Annual cost
Total annual cost
Investment charges
Cost of lost energy
0
100
200
300
400
Weight of conductor in lb. per 1,000 ft.

should be obvious that certain precautions should be taken in its application. For example, the statement that the minimum-cost point occurs when the directly varying costs equal the inversely varying costs is not correct unless the line representing the directly varying costs goes through the origin. Also, the cost represented by ax must actually vary directly with x and the cost represented by $\frac{b}{x}$ must actually vary inversely. In the following example the cost per pound of wire must be the same for all different sizes of wire (which usually is not true), and the costs of energy losses must vary inversely with the wire size. A variable rate for electric energy or the existence of leakage loss and corona loss (such as occur in high-voltage transmission lines) interferes with the second assumption. Moreover, the analysis disregards any possible adverse consequences of voltage drop on the operation of electrical equipment. A lower limit on wire size may exist because of electrical code requirements.

Economical Size of an Electrical Conductor

Facts of the Case

The greater the diameter of an electrical conductor, the less the energy loss that will take place in it. (Power loss in watts is I^2R, where I = current in amperes and R = resistance in ohms. This may be converted to kilowatts by dividing by 1,000. Power loss in kw multiplied by the number of hours it occurs in a given period will give energy loss in kw-hr.) Thus, an increased investment in conductor metal will save an operating expense for electrical energy.

Assume that a conductor is to be selected to carry 50 amperes for 4,200 hours per year, with the cost of wire at $1.75 per pound and electrical energy purchased at 5.5 cents per kw-hr. The life is estimated as 25 years with zero salvage value. The minimum attractive rate of return before income taxes is 14%, and average annual property taxes are estimated at 1.75% of first cost. These charges proportional to investment—namely, capital recovery cost of 14.55% and property taxes of 1.75%—are lumped together as investment charges of 16.3%.

The cross-sectional area of a copper conductor is expressed in circular mils, the weight of the conductor is directly proportional to the cross-sectional area, and the resistance to the flow of current is inversely proportional to the area. Therefore, let x represent the cross-sectional area in circular mils, and x_e represent the most economical size for the stated conditions. The resistance, R, for a conductor of 1,000 ft in length and 1 circular mil in cross-sectional area is approximately 10,580 ohms at 25°C, and the same conductor will weigh approximately 0.00302 lb.

The investment in the conductor will be

The investment in the conductor will be

$$\$1.75(0.00302)x$$

The annual cost will be $\$1.75(0.00302)(0.163)x$. Let

$$\$1.75(0.00302)(0.163) = a = \$0.000861$$

The annual cost of power loss is

$$\frac{I^2R(4{,}200)(\$0.055)}{1{,}000}$$

but

$$R = 10{,}580/x$$

Therefore, the cost of power loss is

$$\frac{(50^2)(4{,}200)(\$0.055)(10{,}580)}{1{,}000x}$$

Let

$$b = \frac{(50^2)(4{,}200)(\$0.055)(10{,}580)}{1{,}000} = \$6{,}109{,}950$$

From the formula developed in the previous article we know that the most economical value of x, x_e, occurs when

$$x = \sqrt{\frac{b}{a}}$$

$$x_e = \sqrt{\frac{\$6{,}109{,}950}{\$0.000861}} = 84{,}240 \text{ circular mils}$$

By examining a table of wire sizes (American Wire Gage, B & S) the closest available conductor is Gage No. 1, with 83,690 circular mils.

Comments on Example 10–4

In this example the size of the conductor was treated as a continuous variable. Actually, the conductors available are discrete sizes, increasing by a geometric progression at the rate of 1.123^2 for the cross-sectional area. The fact that there are specific sizes of wire available and that tables giving the resistance, weight, and cross-sectional area are also available makes another method of solution attractive.

In Table 10–2 five successive wire sizes that might be used for this application have been selected and the annual cost of each size was computed. The table shows that wire size No. 1 gives the lowest annual cost. This table also emphasizes the fact that one type of cost increases as the other decreases, as happens in many problems involving multiple alternatives. Furthermore, note that at the most economical alternative the investment charges approximately equal the cost of lost energy. This is characteristic of problems in which the total costs can be represented by the equation

$$y = ax + \frac{b}{x} + c$$

It was first pointed out by Lord Kelvin in 1881 that the economical size of conductor is that for which the annual investment charges just equal the annual cost of lost energy. This is well known in electrical engineering as Kelvin's Law. The costs in Figure 10–1 illustrate its application.

Some Comments about the Use of Mathematical Models in Engineering Economy

Many mathematical models have been developed to guide decisions among investment alternatives. General statements may be made that are applicable to many

TABLE 10–2

Comparison of annual costs of various wire sizes in the selection of an electrical conductor

(Investment in wire at $1.75/lb; current of 50 amperes for 4,200 hrs/yr; all calculations based on 1,000 ft of copper wire)

A. Size of wire (AWG)	00	0	1	2	3
B. Weight of wire in lb	403	319	253	201	159
C. Investment in wire	$705.25	$558.25	$442.75	$351.75	$278.25
D. Resistance in ohms	0.0795	0.100	0.126	0.159	0.201
E. Power loss in kw	0.1987	0.250	0.315	0.398	0.503
F. Annual energy loss in kw-hr	835	1,050	1,323	1,670	2,111
G. Investment charges at 16.3%	$114.96	$90.99	$72.17	$57.34	$45.35
H. Cost of lost energy at 5.5¢ kw-hr	45.93	57.75	72.77	91.85	116.11
I. Total annual cost assumed to be variable with wire size	$160.89	$148.74	$144.94	$149.19	$161.46

The long-run profitability of the enterprise hinges on the solution of two problems of management of corporate capital: (1) sourcing (acquisition) of capital funds and (2) rationing (investment) of that capital. They should be quite separate. Investment proposals should compete for corporate funds on the basis of financial merit (the productivity of capital), independent of the source or cost of funds for that particular project. Investable funds of the corporation should be treated as a common pool, not compartmented puddles. Similarly, the problem of acquiring capital should be solved independent of its rationing and also on the basis of merit (the comparative costs and risks of alternative patterns of sourcing).—Joel Dean[1]

Factors Considered in Setting i^*

The factors usually considered in the determination of the i^* to be used during any period of time include:

1. Availability of funds for investment and their sources—equity or borrowing.
2. Competing investment opportunities.
3. Differences in the risk involved in the different competing investment opportunities.
4. Differences in the time required for recovery of the investment with the desired rate of return—short-lived versus long-lived investments.
5. The "going price of money" as represented by the interest rates paid or charged on such investments as FDIC-insured savings accounts, the "prime rate" used by large banks, and the government short- and long-term notes and bonds.
6. Analysis before or after income taxes.

TABLE 14-1
United States Consumer Price Index, 1913-1987

Year	CPI	Year	CPI	Year	CPI
1913	9.9	1938	14.1	1963	30.6
1914	10.0	1939	13.9	1964	31.0
1915	10.1	1940	14.0	1965	31.5
1916	10.9	1941	14.7	1966	32.4
1917	12.8	1942	16.3	1967	33.4
1918	15.1	1943	17.3	1968	34.8
1919	17.3	1944	17.6	1969	36.7
1920	20.0	1945	18.0	1970	38.8
1921	17.9	1946	19.5	1971	40.5
1922	16.8	1947	22.3	1972	41.8
1923	17.1	1948	24.1	1973	44.4
1924	17.1	1949	23.8	1974	49.3
1925	17.5	1950	24.1	1975	53.8
1926	16.7	1951	26.0	1976	56.9
1927	17.4	1952	26.5	1977	60.6
1928	17.1	1953	26.7	1978	65.2
1929	17.1	1954	26.9	1979	72.6
1930	16.7	1955	26.8	1980	82.5
1931	15.2	1956	27.2	1981	90.9
1932	13.7	1957	28.1	1982	96.5
1933	13.0	1958	28.9	1983	99.6
1934	13.4	1959	29.1	1984	103.9
1935	13.7	1960	29.6	1985	107.6
1936	13.9	1961	29.9	1986	109.6
1937	14.4	1962	30.2	1987	113.6

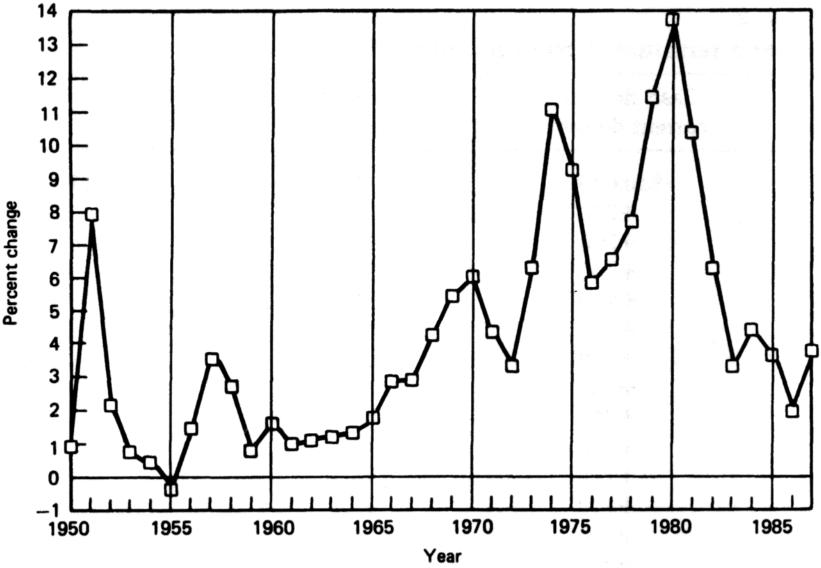


FIGURE 14–1 Annual percentage change in United States Consumer Price Index.

15–10. (This is adapted from an example in a paper by C. H. Oglesby and E. L. Grant published in Volume 37 of *Highway Research Board Proceedings*.)

It is desired to select a size for a box culvert for a rural highway in central Illinois. The drainage area has 400 acres of mixed cover with slopes greater than 2%. The culvert will be 200 ft long. Because headroom is critical, the culvert can be only 4 ft high. If the water rises more than 5 ft above the streambed, the road will be overtopped. Damage to highway and adjacent property for each overtopping will be $150,000.

If the project is to be built at all, the minimum acceptable culvert for this location is a simple box 10 × 4 ft, which will be overtopped, on the average, once in 5 years. Four possible designs, with associated initial costs, capacities, and probabilities of overtopping, are:

Combination	3,600,000
Oil	4,500,000

Draw a decision tree to describe the alternatives and probable events case. If the company makes its decision based on maximum expected value alternative will be chosen? Discuss the various ramifications of the outcome and probabilities associated with this decision.

15–13. Set up a decision matrix as described in the text for Problem 15–1 the maximum security level strategy. What are the implications of a strategy type applied to decisions of the type faced by a small independent oil company such as Ewing Oil? Discuss this question in relationship to frequency probabilities, as used in Problem 15–12, and intuitive probabil

	First Cost	Capacity cu ft/sec	Probability of Overtopping in Any One Year
A. Single culvert 10 × 4 ft	\$162,000	300	0.20
B. Double culvert 8 × 4 ft	\$210,000	400	0.10
C. Double culvert 10 × 4 ft	\$250,000	500	0.04
D. Triple culvert 8 × 4 ft	\$310,000	600	0.02

Which design gives the lowest sum of the annual cost of capital recovery of the investment and the expected value of the annual damage from overtopping? Assume an i^* of 10%. Assume culverts will have lives of 50 years with zero salvage value.

EXAMPLE 16–2

An Unsound Analysis of a Public Works Proposal

Facts of the Case

A certain consulting firm was employed to make a benefit–cost analysis of a proposed county expressway project. The conclusion was that annual benefits would be $400,000 and annual costs would be $1,000,000, yielding a **B/C** ratio of 0.4. On this basis, the project did not appear to be justified.

However, the firm had a bright idea. They proposed that the supervisors make an effort to have this expressway incorporated into the interstate highway system. If this could be done, 90% of the cost would be paid by the federal government. They advised the supervisors that this would reduce the local annual costs to $100,000 and that the **B/C** ratio would then be $400,000 ÷ $100,000 = 4.0.

"American society experienced a virtual explosion in Government regulation during the past decades. Between 1970 and 1979, expenditures for the major regulatory agencies quadrupled, the number of pages published annually in the Federal Register nearly tripled and the number of pages in the Code of Federal Regulations increased by nearly two-thirds.

'The result has been higher prices, higher unemployment and lower productivity growth. Over-regulation causes small and independent businessmen and women, as well as large businesses, to defer or terminate plans for expansion and, since they are responsible for most of our new jobs, those jobs aren't created.

'We have no intention of dismantling the regulatory agencies—especially those necessary to protect the environment and to assure public health and safety. However, we must come to grips with inefficient and burdensome regulations—eliminate those we can and reform those we must keep."[1]

How Large Is the Cost of Regulation

Previously we noted the difficulty of estimating benefits to be derived from compliance with specific regulations. On a nationwide basis it is almost as difficult to estimate the costs to the citizens. Every report, document, or record adds to the clerical and overhead costs of doing business. Huge, almost unbelievable quantities of paper containing the required information are prepared each month for national, state, and local governments. Thousands of persons are employed each day in collecting, analyzing, and preparing data for these documents. Similarly, thousands of government employees spend their time reviewing these documents, yet this is only one aspect of the total cost of regulation.

At various times different individuals and agencies have made estimates of the total cost of regulation. These estimates have varied from $15 to $20 billion to as much as $100 billion a year. These astronomical numbers illustrate why both business and government ought to examine very carefully both the costs and benefits of any specific set of regulations *before* reaching a decision to impose it.

D

COMPOUND INTEREST TABLES

Formulas for Calculating Compound Interest Factors

Single Payment—Compound Amount Factor
(*F/P, i%, n*)

$$(1+i)^n$$

Single Payment—Present Worth Factor
(*P/F, i%, n*)

$$\frac{1}{(1+i)^n}$$

Sinking Fund Factor
(*A/F, i%, n*)

$$\frac{i}{(1+i)^n-1}$$

Capital Recovery Factor
(*A/P, i%, n*)

$$\frac{i(1+i)^n}{(1+i)^n-1}$$

Uniform Series—Compound Amount Factor
(*F/A, i%, n*)

$$\frac{(1+i)^n-1}{i}$$

Uniform Series—Present Worth Factor
(*P/A, i%, n*)

$$\frac{(1+i)^n-1}{i(1+i)^n}$$

Uniform Gradient—Conversion Factor
(*A/G, i%, n*)

$$\frac{1}{i}-\frac{n}{i}\left[\frac{i}{(1+i)^n-1}\right]$$

Uniform Gradient—Present Worth Factor
(*P/G, i%, n*)

$$\frac{1}{i}\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]-\frac{n}{i(1+i)^n}$$

TABLE D-9
5% compound interest factors

	Single Payment		Uniform Series				Uniform Gradient		
n	Compound amount factor F/P	Present worth factor P/F	Sinking fund factor A/F	Capital recovery factor A/P	Compound amount factor F/A	Present worth factor P/A	Gradient conversion factor A/G	Present worth factor P/G	n
1	1.0500	0.9524	1.000 00	1.050 00	1.000	0.952	0.000	0.000	1
2	1.1025	0.9070	0.487 80	0.537 80	2.050	1.859	0.488	0.907	2
3	1.1576	0.8638	0.317 21	0.367 21	3.153	2.723	0.967	2.635	3
4	1.2155	0.8227	0.232 01	0.282 01	4.310	3.546	1.439	5.103	4
5	1.2763	0.7835	0.180 97	0.230 97	5.526	4.329	1.903	8.237	5
6	1.3401	0.7462	0.147 02	0.197 02	6.802	5.076	2.358	11.968	6
7	1.4071	0.7107	0.122 82	0.172 82	8.142	5.786	2.805	16.232	7
8	1.4775	0.6768	0.104 72	0.154 72	9.549	6.463	3.245	20.970	8
9	1.5513	0.6446	0.090 69	0.140 69	11.027	7.108	3.676	26.127	9
10	1.6289	0.6139	0.079 50	0.129 50	12.578	7.722	4.099	31.652	10
11	1.7103	0.5847	0.070 39	0.120 39	14.207	8.306	4.514	37.499	11
12	1.7959	0.5568	0.062 83	0.112 83	15.917	8.863	4.922	43.624	12
13	1.8856	0.5303	0.056 46	0.106 46	17.713	9.394	5.322	49.988	13
14	1.9800	0.5051	0.051 02	0.101 02	19.599	9.899	5.713	56.554	14
15	2.0789	0.4810	0.046 34	0.096 34	21.579	10.380	6.097	63.288	15
16	2.1829	0.4581	0.042 27	0.092 27	23.657	10.838	6.474	70.160	16
17	2.2920	0.4363	0.038 70	0.088 70	25.840	11.274	6.842	77.140	17
18	2.4066	0.4155	0.035 55	0.085 55	28.132	11.690	7.203	84.204	18
19	2.5270	0.3957	0.032 75	0.082 75	30.539	12.085	7.557	91.328	19
20	2.6533	0.3769	0.030 24	0.080 24	33.066	12.462	7.903	98.488	20
21	2.7860	0.3589	0.028 00	0.078 00	35.719	12.821	8.242	105.667	21
22	2.9253	0.3418	0.025 97	0.075 97	38.505	13.163	8.573	112.846	22
23	3.0715	0.3256	0.024 14	0.074 14	41.430	13.489	8.897	120.009	23
24	3.2251	0.3101	0.022 47	0.072 47	44.502	13.799	9.214	127.140	24
25	3.3864	0.2953	0.020 95	0.070 95	47.727	14.094	9.524	134.228	25
26	3.5557	0.2812	0.019 56	0.069 56	51.113	14.375	9.827	141.259	26
27	3.7335	0.2678	0.018 29	0.068 29	54.669	14.643	10.122	148.223	27
28	3.9201	0.2551	0.017 12	0.067 12	58.403	14.898	10.411	155.110	28
29	4.1161	0.2429	0.016 05	0.066 05	62.323	15.141	10.694	161.913	29
30	4.3219	0.2314	0.015 05	0.065 05	66.439	15.372	10.969	168.623	30
31	4.5380	0.2204	0.014 13	0.064 13	70.761	15.593	11.238	175.233	31
32	4.7649	0.2099	0.013 28	0.063 28	75.299	15.803	11.501	181.739	32
33	5.0032	0.1999	0.012 49	0.062 49	80.064	16.003	11.757	188.135	33
34	5.2533	0.1904	0.011 76	0.061 76	85.067	16.193	12.006	194.417	34
35	5.5160	0.1813	0.011 07	0.061 07	90.320	16.374	12.250	200.581	35
40	7.0400	0.1420	0.008 28	0.058 28	120.800	17.159	13.377	229.545	40
45	8.9850	0.1113	0.006 26	0.056 26	159.700	17.774	14.364	255.315	45
50	11.4674	0.0872	0.004 78	0.054 78	209.348	18.256	15.223	277.915	50
55	14.6356	0.0683	0.003 67	0.053 67	272.713	18.633	15.966	297.510	55
60	18.6792	0.0535	0.002 83	0.052 83	353.584	18.929	16.606	314.343	60
65	23.8399	0.0419	0.002 19	0.052 19	456.798	19.161	17.154	328.691	65
70	30.4264	0.0329	0.001 70	0.051 70	588.529	19.343	17.621	340.841	70
75	38.8327	0.0258	0.001 32	0.051 32	756.654	19.485	18.018	351.072	75
80	49.5614	0.0202	0.001 03	0.051 03	971.229	19.596	18.353	359.646	80
85	63.2544	0.0158	0.000 80	0.050 80	1 245.087	19.684	18.635	366.801	85
90	80.7304	0.0124	0.000 63	0.050 63	1 594.607	19.752	18.871	372.749	90
95	103.0357	0.0097	0.000 49	0.050 49	2 040.694	19.806	19.069	377.677	95
100	131.5013	0.0076	0.000 38	0.050 38	2 610.025	19.848	19.234	381.749	100

TABLE D-11
6% compound interest factors

	Single Payment		Uniform Series				Uniform Gradient		
n	Compound amount factor *F/P*	Present worth factor *P/F*	Sinking fund factor *A/F*	Capital recovery factor *A/P*	Compound amount factor *F/A*	Present worth factor *P/A*	Gradient conversion factor *A/G*	Present worth factor *P/G*	*n*
1	1.0600	0.9434	1.000 00	1.060 00	1.000	0.943	0.000	0.000	1
2	1.1236	0.8900	0.485 44	0.545 44	2.060	1.833	0.485	0.890	2
3	1.1910	0.8396	0.314 11	0.374 11	3.184	2.673	0.961	2.569	3
4	1.2625	0.7921	0.228 59	0.288 59	4.375	3.465	1.427	4.946	4
5	1.3382	0.7473	0.177 40	0.237 40	5.637	4.212	1.884	7.935	5
6	1.4185	0.7050	0.143 36	0.203 36	6.975	4.917	2.330	11.459	6
7	1.5036	0.6651	0.119 14	0.179 14	8.394	5.582	2.768	15.450	7
8	1.5938	0.6274	0.101 04	0.161 04	9.897	6.210	3.195	19.842	8
9	1.6895	0.5919	0.087 02	0.147 02	11.491	6.802	3.613	24.577	9
10	1.7908	0.5584	0.075 87	0.135 87	13.181	7.360	4.022	29.602	10
11	1.8983	0.5268	0.066 79	0.126 79	14.972	7.887	4.421	34.870	11
12	2.0122	0.4970	0.059 28	0.119 28	16.870	8.384	4.811	40.337	12

TABLE D-12
7% compound interest factors

	Single Payment		Uniform Series				Uniform Gradient		
n	Compound amount factor F/P	Present worth factor P/F	Sinking fund factor A/F	Capital recovery factor A/P	Compound amount factor F/A	Present worth factor P/A	Gradient conversion factor A/G	Present worth factor P/G	n
1	1.0700	0.9346	1.000 00	1.070 00	1.000	0.935	0.000	0.000	1
2	1.1449	0.8734	0.483 09	0.553 09	2.070	1.808	0.483	0.873	2
3	1.2250	0.8163	0.311 05	0.381 05	3.215	2.624	0.955	2.506	3
4	1.3108	0.7629	0.225 23	0.295 23	4.440	3.387	1.416	4.795	4
5	1.4026	0.7130	0.173 89	0.243 89	5.751	4.100	1.865	7.647	5
6	1.5007	0.6663	0.139 80	0.209 80	7.153	4.767	2.303	10.978	6
7	1.6058	0.6227	0.115 55	0.185 55	8.654	5.389	2.730	14.715	7
8	1.7182	0.5820	0.097 47	0.167 47	10.260	5.971	3.147	18.789	8
9	1.8385	0.5439	0.083 49	0.153 49	11.978	6.515	3.552	23.140	9
10	1.9672	0.5083	0.072 38	0.142 38	13.816	7.024	3.946	27.716	10
11	2.1049	0.4751	0.063 36	0.133 36	15.784	7.499	4.330	32.466	11
12	2.2522	0.4440	0.055 90	0.125 90	17.888	7.943	4.703	37.351	12
13	2.4098	0.4150	0.049 65	0.119 65	20.141	8.358	5.065	42.330	13
14	2.5785	0.3878	0.044 34	0.114 34	22.550	8.745	5.417	47.372	14
15	2.7590	0.3624	0.039 79	0.109 79	25.129	9.108	5.758	52.446	15
16	2.9522	0.3387	0.035 86	0.105 86	27.888	9.447	6.090	57.527	16
17	3.1588	0.3166	0.032 43	0.102 43	30.840	9.763	6.411	62.592	17
18	3.3799	0.2959	0.029 41	0.099 41	33.999	10.059	6.722	67.622	18
19	3.6165	0.2765	0.026 75	0.096 75	37.379	10.336	7.024	72.599	19
20	3.8697	0.2584	0.024 39	0.094 39	40.995	10.594	7.316	77.509	20
21	4.1406	0.2415	0.022 29	0.092 29	44.865	10.836	7.599	82.339	21
22	4.4304	0.2257	0.020 41	0.090 41	49.006	11.061	7.872	87.079	22
23	4.7405	0.2109	0.018 71	0.088 71	53.436	11.272	8.137	91.720	23
24	5.0724	0.1971	0.017 19	0.087 19	58.177	11.469	8.392	96.255	24
25	5.4274	0.1842	0.015 81	0.085 81	63.249	11.654	8.639	100.676	25
26	5.8074	0.1722	0.014 56	0.084 56	68.676	11.826	8.877	104.981	26
27	6.2139	0.1609	0.013 43	0.083 43	74.484	11.987	9.107	109.166	27
28	6.6488	0.1504	0.012 39	0.082 39	80.698	12.137	9.329	113.226	28
29	7.1143	0.1406	0.011 45	0.081 45	87.347	12.278	9.543	117.162	29
30	7.6123	0.1314	0.010 59	0.080 59	94.461	12.409	9.749	120.972	30
31	8.1451	0.1228	0.009 80	0.079 80	102.073	12.532	9.947	124.655	31
32	8.7153	0.1147	0.009 07	0.079 07	110.218	12.647	10.138	128.212	32
33	9.3253	0.1072	0.008 41	0.078 41	118.933	12.754	10.322	131.643	33
34	9.9781	0.1002	0.007 80	0.077 80	128.259	12.854	10.499	134.951	34
35	10.6766	0.0937	0.007 23	0.077 23	138.237	12.948	10.669	138.135	35
40	14.9745	0.0668	0.005 01	0.075 01	199.635	13.332	11.423	152.293	40
45	21.0025	0.0476	0.003 50	0.073 50	285.749	13.606	12.036	163.756	45
50	29.4570	0.0339	0.002 46	0.072 46	406.529	13.801	12.529	172.905	50
55	41.3150	0.0242	0.001 74	0.071 74	575.929	13.940	12.921	180.124	55
60	57.9464	0.0173	0.001 23	0.071 23	813.520	14.039	13.232	185.768	60
65	81.2729	0.0123	0.000 87	0.070 87	1 146.755	14.110	13.476	190.145	65
70	113.9894	0.0088	0.000 62	0.070 62	1 614.134	14.160	13.666	193.519	70
75	159.8760	0.0063	0.000 44	0.070 44	2 269.657	14.196	13.814	196.104	75
80	224.2344	0.0045	0.000 31	0.070 31	3 189.063	14.222	13.927	198.075	80
85	314.5003	0.0032	0.000 22	0.070 22	4 478.576	14.240	14.015	199.572	85
90	441.1030	0.0023	0.000 16	0.070 16	6 287.185	14.253	14.081	200.704	90
95	618.6697	0.0016	0.000 11	0.070 11	8 823.854	14.263	14.132	201.558	95
100	867.7163	0.0012	0.000 08	0.070 08	12 381.662	14.269	14.170	202.200	100

TABLE D-15
10% compound interest factors

	Single Payment		Uniform Series				Uniform Gradient		
n	Compound amount factor F/P	Present worth factor P/F	Sinking fund factor A/F	Capital recovery factor A/P	Compound amount factor F/A	Present worth factor P/A	Gradient conversion factor A/G	Present worth factor P/G	n
1	1.1000	0.9091	1.000 00	1.100 00	1.000	0.909	0.000	0.000	1
2	1.2100	0.8264	0.476 19	0.576 19	2.100	1.736	0.476	0.826	2
3	1.3310	0.7513	0.302 11	0.402 11	3.310	2.487	0.937	2.329	3
4	1.4641	0.6830	0.215 47	0.315 47	4.641	3.170	1.381	4.378	4
5	1.6105	0.6209	0.163 80	0.263 80	6.105	3.791	1.810	6.862	5
6	1.7716	0.5645	0.129 61	0.229 61	7.716	4.355	2.224	9.684	6
7	1.9487	0.5132	0.105 41	0.205 41	9.487	4.868	2.622	12.763	7
8	2.1436	0.4665	0.087 44	0.187 44	11.436	5.335	3.004	16.029	8
9	2.3579	0.4241	0.073 64	0.173 64	13.579	5.759	3.372	19.421	9
10	2.5937	0.3855	0.062 75	0.162 75	15.937	6.144	3.725	22.891	10
11	2.8531	0.3505	0.053 96	0.153 96	18.531	6.495	4.064	26.396	11
12	3.1384	0.3186	0.046 76	0.146 76	21.384	6.814	4.388	29.901	12
13	3.4523	0.2897	0.040 78	0.140 78	24.523	7.103	4.699	33.377	13
14	3.7975	0.2633	0.035 75	0.135 75	27.975	7.367	4.996	36.800	14
15	4.1772	0.2394	0.031 47	0.131 47	31.772	7.606	5.279	40.152	15
16	4.5950	0.2176	0.027 82	0.127 82	35.950	7.824	5.549	43.416	16
17	5.0545	0.1978	0.024 66	0.124 66	40.545	8.022	5.807	46.582	17
18	5.5599	0.1799	0.021 93	0.121 93	45.599	8.201	6.053	49.640	18
19	6.1159	0.1635	0.019 55	0.119 55	51.159	8.365	6.286	52.583	19
20	6.7275	0.1486	0.017 46	0.117 46	57.275	8.514	6.508	55.407	20
21	7.4002	0.1351	0.015 62	0.115 62	64.002	8.649	6.719	58.110	21
22	8.1403	0.1228	0.014 01	0.114 01	71.403	8.772	6.919	60.689	22
23	8.9543	0.1117	0.012 57	0.112 57	79.543	8.883	7.108	63.146	23
24	9.8497	0.1015	0.011 30	0.111 30	88.497	8.985	7.288	65.481	24
25	10.8347	0.0923	0.010 17	0.110 17	98.347	9.077	7.458	67.696	25
26	11.9182	0.0839	0.009 16	0.109 16	109.182	9.161	7.619	69.794	26
27	13.1100	0.0763	0.008 26	0.108 26	121.100	9.237	7.770	71.777	27
28	14.4210	0.0693	0.007 45	0.107 45	134.210	9.307	7.914	73.650	28
29	15.8631	0.0630	0.006 73	0.106 73	148.631	9.370	8.049	75.415	29
30	17.4494	0.0573	0.006 08	0.106 08	164.494	9.427	8.176	77.077	30
31	19.1943	0.0521	0.005 50	0.105 50	181.943	9.479	8.296	78.640	31
32	21.1138	0.0474	0.004 97	0.104 97	201.138	9.526	8.409	80.108	32
33	23.2252	0.0431	0.004 50	0.104 50	222.252	9.569	8.515	81.486	33
34	25.5477	0.0391	0.004 07	0.104 07	245.477	9.609	8.615	82.777	34
35	28.1024	0.0356	0.003 69	0.103 69	271.024	9.644	8.709	83.987	35
40	45.2593	0.0221	0.002 26	0.102 26	442.593	9.779	9.096	88.953	40
45	72.8905	0.0137	0.001 39	0.101 39	718.905	9.863	9.374	92.454	45
50	117.3909	0.0085	0.000 86	0.100 86	1 163.909	9.915	9.570	94.889	50
55	189.0591	0.0053	0.000 53	0.100 53	1 880.591	9.947	9.708	96.562	55
60	304.4816	0.0033	0.000 33	0.100 33	3 034.816	9.967	9.802	97.701	60
65	490.3707	0.0020	0.000 20	0.100 20	4 893.707	9.980	9.867	98.471	65
70	789.7470	0.0013	0.000 13	0.100 13	7 887.470	9.987	9.911	98.987	70
75	1 271.8952	0.0008	0.000 08	0.100 08	12 708.954	9.992	9.941	99.332	75
80	2 048.4002	0.0005	0.000 05	0.100 05	20 474.002	9.995	9.961	99.561	80
85	3 298.9690	0.0003	0.000 03	0.100 03	32 979.690	9.997	9.974	99.712	85
90	5 313.0226	0.0002	0.000 02	0.100 02	53 120.226	9.998	9.983	99.812	90
95	8 556.6760	0.0001	0.000 01	0.100 01	85 556.760	9.999	9.989	99.877	95
100	13 780.6123	0.0001	0.000 01	0.100 01	137 796.123	9.999	9.993	99.920	100